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Chiral condensate thermal evolution at finite baryon chemical potential within ChPT

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Abstract. We present a model independent study of the chiral condensate evolution in a hadronic gas, in terms of temperature and baryon chemical potential. The meson-meson interactions are described within Chiral Perturbation Theory and the pion-nucleon interaction by means of Heavy Baryon Chiral Perturbation Theory, both at one loop. Together with the virial expansion, this provides a model independent systematic expansion at low temperatures and chemical potentials, which includes the physical quark masses. This can serve as a guideline for further studies on the lattice. We also obtain estimates of the critical line of temperature and chemical potential where the chiral condensate melts, which systematically lie somewhat higher than recent lattice calculations but are consistent with several hadronic models.

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On a recent work [1] we approach the question of the phase diagram of QCD. In particular, we study the transition from the hadronic phase, in which the chiral symmetry is spontaneously broken, to a phase in which it is restored. This is done within Chiral Perturbation Theory (ChPT), the low energy effective theory of QCD. We also gave estimates for the melting curve of the quark condensate in presence of a baryon chemical potential.

Let us briefly describe ChPT. The spontaneous chiral symmetry breaking of QCD requires the existence of eight massless Goldstone Bosons (GB), that can be identified with the pions, kaons and eta. These are thus the most relevant degrees of freedom at low energies. In addition, there is an explicit symmetry breaking due to the non-vanishing quark masses that give rise to a small mass for the pions, kaons and eta, which are thus just pseudo-GB. ChPT is built as the most general derivative and mass expansion over the spontaneous symmetry breaking (SSB) scale $\Lambda_\chi \equiv 4\pi f_\pi \simeq 1.2$ GeV which respects the symmetry constraints of QCD. Baryons can be included in this formalism, although its treatment is more involved due to their large masses. Within Heavy Baryon ChPT (HBChPT) [3] this problem is overcome for the meson-baryon interaction by an additional expansion on inverse powers of the baryon mass. This approach has proven very successful, and works remarkably well within the meson sector, and with a somewhat slower convergence in the meson-baryon sector.

Next, the thermodynamics of the hadron gas is described by the virial expansion [4,5]. This is a low density series expansion for the grand canonical potential of a relativistic interacting multicomponent gas. We will assume that only the strong interactions are relevant, and that the baryon density defined as $n_B - n_{\bar{B}}$ is conserved. The thermodynamics of the gas will thus depend on the temperature T (usually written in terms of its inverse $\beta = 1/T$) and a baryon chemical potential μ_B. The virial expansion, written in terms of the pressure, reads:

$$\beta P = \sum_i B_i^{(1)}\,\xi_i + B_i^{(2)}\,\xi_i^2 + \sum_i \sum_{j\geq i} B_{ij}^{int}\,\xi_i\xi_j + \dots, \quad (1)$$

where $\xi_i = \exp\beta(\mu_i - M_i)$, with M_i the mass of the i-th species, $\mu_i = 0, \pm\mu_B$ for mesons/(anti)baryons respectively. The free virial coefficients

$$B_i^{(n)} = \frac{g_i \eta_i^{n+1}}{2\pi^2}\int_0^\infty dp\,p^2\,e^{-n\beta(\sqrt{p^2+M_i^2}-M_i)}, \quad (2)$$

, with $\eta_i = \pm 1$ for bosons/mesons and g_i the degeneracy of the i-th species, correspond simply to the series expansion of the pressure for a free gas, while the interactions are taken into account in the term:

$$B_{ij}^{int} = \frac{e^{\beta(M_i+M_j)}}{2\,\pi^3}\int_{M_i+M_j}^\infty dE\,E^2 K_1(\beta E)\Delta^{ij}(E), \quad (3)$$

where $K_1(x)$ is the modified Bessel function (of the second kind), and $\Delta^{ij} = \sum_{I,J,S}(2I+1)(2J+1)\delta^{ij}$ are the $ij \to ij$ elastic scattering phase shifts for a state ij with well defined isospin I, total angular momentum J and strangeness S, defined so that $\delta = 0$ at threshold.

The dependence of the chiral condensate with temperature and chemical potential is written in terms of the pressure of the gas as follows:

$$\langle\bar{q}q\rangle = \langle 0|\bar{q}q|0\rangle\left(1 + \sum_h \frac{c_h}{2M_hF^2}\frac{\partial P}{\partial M_h}\right), \quad (4)$$

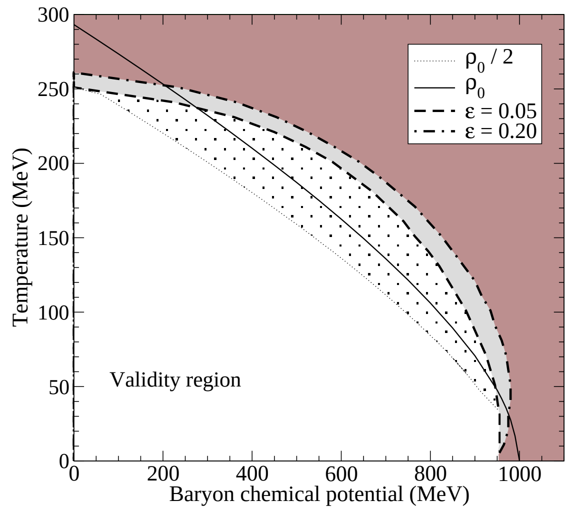

Fig. 1. Crude estimate of the region of validity of the virial expansion for a free gas. We have excluded the regions in which the virial expansion breaks, but also those regions in which nuclear density is large enough so that higher order effects are relevant.

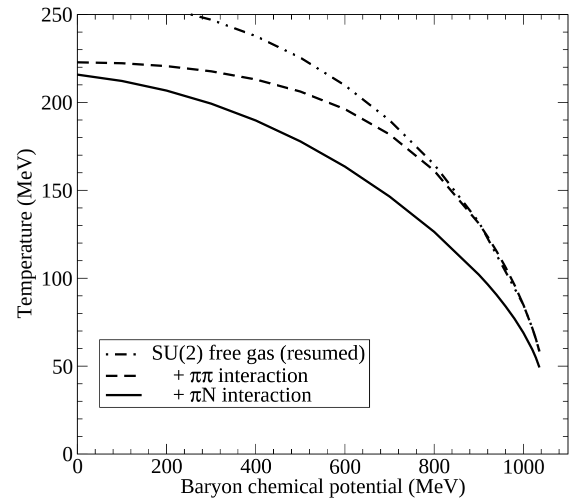

Fig. 2. Melting line in the (μ_B, T) plane of the chiral condensate for an SU(2) gas of hadrons. We show the result for the free gas, but also adding the $\pi\pi$ ChPT SU(2) interaction to one loop, and the line resulting from adding the πN interaction to third order in HBChPT.

where the constant F, which is the pion decay constant in the chiral limit, is introduced for further convenience, and the coefficients $c_h = -F^2 \frac{\partial M_h^2}{\partial \hat{m}} \langle 0|\bar{q}q|0\rangle^{-1}$ encode the hadron mass dependence on the quark mass. Numerically, these coefficients amount to $c_\pi = 0.9^{+0.2}_{-0.4}$, $c_K = 0.5^{+0.4}_{-0.7}$, $c_\eta = 0.4^{+0.5}_{-0.7}$, for mesons [6], and $c_N = 3.6^{+1.5}_{-1.9}$ for nucleons [1].

First of all, we want to estimate the region of validity of our calculations. For this, we check the points for which the virial expansion breaks down, by computing the difference between the free condensate calculated exactly and with the virial expansion. In Figure 1 we show, as a dashed and a dashed-dotted lines, the points where the virial expansion is off the exact calculation by 5% and 20%, respectively. The expansion rapidly deteriorates after that. Also, as we want to neglect NN interactions, we have to restrict ourselves to the region where nucleon density is small enough. In ref. [7] it is shown that, for densities below $\rho_0/2$, with $\rho_0 \simeq 0.16\,\mathrm{fm}^{-3}$, and *as far as we are only concerned with the quark condensate*, the πN interaction dominates over the NN interaction. Thus, we restrict the validity of our calculation to the points below the $\rho_0/2$ line. This line is shown in the Figure as a dotted line, together with the ρ_0 line, which is shown for illustration as a continuous line. The white area in Fig.2 corresponds to our estimated "validity region".

In Figure 2 we show in the (μ_B, T) plane the condensate melting line of the free $SU(2)$ gas composed of pions and nucleons as a dash-dot-dotted line. We have then added the effects of the $\pi\pi$ interaction, which yields the dashed line of the Figure. Finally, the πN interaction was included. We see that the effect of the πN interaction is small, but noticeable (~ 6 MeV at $\mu_B = 0$) at low baryon chemical potentials, and maximum at around $\mu_B \simeq 600 - 700$ MeV, where it produces a decrease in the melting temperature of about 40 MeV with respect to the gas without πN interactions. Up to a chemical potential of $\mu_B = 40 - 50$ MeV, the region of relevance for Relativistic Heavy Ion Collisions, the decrease in the melting temperature amounts roughly to 10 MeV when we include the πN interaction. It is important to note that, although the critical lines are actually extrapolations, at low temperature and chemical potential our calculation is model independent, and should be quite accurate. Still, we show the melting lines since they are useful to visualize and help quantifying the relative size of each contribution we add into the gas.

In a real gas, we should consider all hadrons. We will do this by including them as free particles. The only exception to this will be the kaons and etas, which are abundant enough up to 200 MeV to deserve a separate treatment and include their interaction with a pion. All other interactions are suppressed by Boltzmann (thermal) and c_h/M_h factors, and are therefore not included in our treatment. At very large μ_B the heavier nucleons may not be Boltzmann suppressed, and their interactions may become important, but this is outside the scope of this work. In Figure 3 we show the two lines corresponding to the $SU(2)$ gas with $\pi\pi$ interactions (dashed) and with $\pi\pi$ and πN interactions (continuous). We then plot the line corresponding to a gas in which also kaons, etas, and their interactions with a pion, now in $SU(3)$, are included (dotted line). Note that the effect is pronounced at $\mu_B = 0$, with a decrease of the melting temperature of around ~ 13 MeV, but becomes negligible at very high baryon chemical potential. Finally, we have added the effects of other heavier hadrons, included as free particles, since they suffer a larger Boltzmann suppression. We have estimated that, for heavier hadrons, $\partial M_h/\partial \hat{m} \simeq \alpha N^h_{u,d}$, where α is an adi-

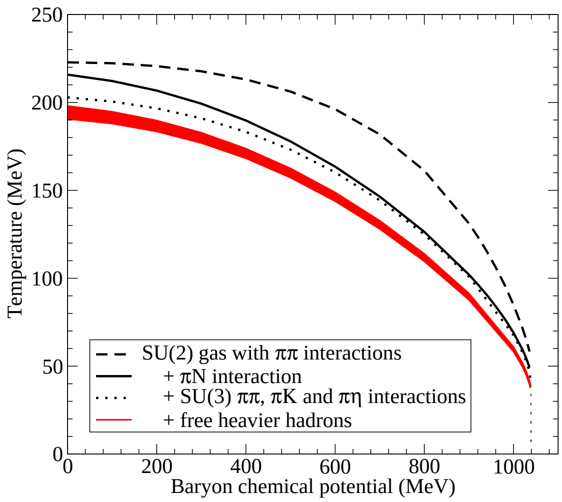

Fig. 3. Our final result for the chiral condensate melting line in the (μ_B, T) plane. Starting from the SU(2) gas with $\pi\pi$ interactions, we show the effect of πN interactions and of adding kaons and etas. The dark area covers the uncertainty due to heavier hadrons estimated as explained in the text.

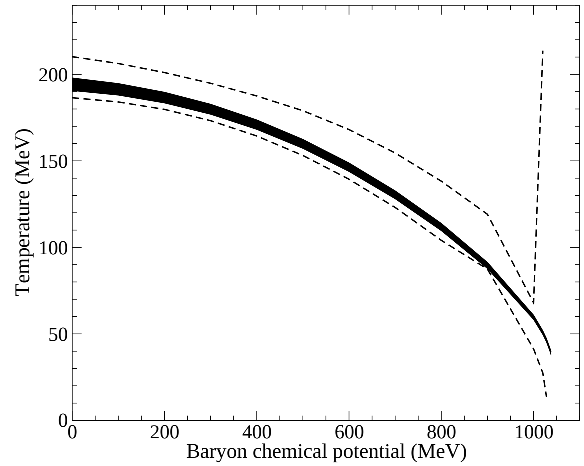

Fig. 4. Analysis of uncertainties. To the dark area, which corresponds to the case described in Fig. 3, we have added the uncertainties coming from the chiral parameters, shown as dashed lines (see [1] for discussion).

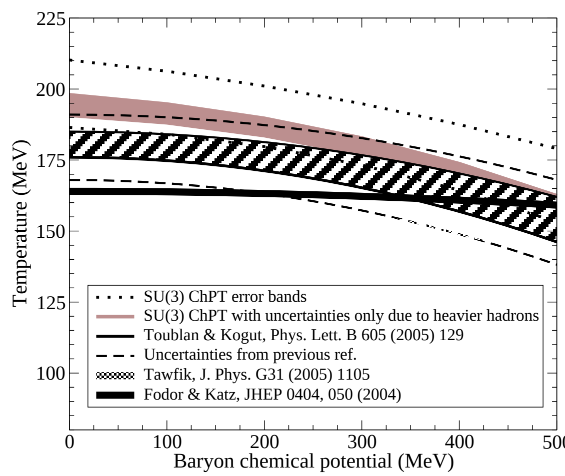

Fig. 5. Comparison between our results and others in the literature. Our melting temperatures are consistent within errors with ref. [8] but systematically higher than [9].

mensional constant and $N^h_{u,d}$ is the number of valence u or d quarks in the hadron. Based on the values of α that we have calculated for the pions, kaons, etas and nucleons, we have estimated that, for heavier hadrons, $\alpha \simeq 0.5 - 2.5$. The band in Figure 3 corresponds to this range of values for α. In order to properly analyse the uncertainties in our calculation, we also have to include the uncertainties that come from our imperfect knowledge of the chiral parameters. This was done by adding in quadrature the errors coming from independently varying each of the parameters. The final results are plotted in Figure 4, where the dark band corresponds to the uncertainties coming from the α parameter in heavy hadrons and the dashed line includes also the uncertainty coming from the chiral parameters. Note that the error is highly asymmetric: even though every contribution lowers the melting temperature, the melting of the condensate accelerates near the melting point, and thus even if the contribution has a similar size to the previous one, its effect on the condensate melting seems smaller.

It is interesting to compare our results with those obtained by using lattice calculations. In Figure 5 we show our curves with their errors, plotted against other results recently appeared in the literature. Our approach agrees, within errors, with the hadron resonance gas model described in [8]. However, ChPT calculations [5,6], including ours [1], yield melting temperatures that are systematically above the results from the lattice calculations [9].

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